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SEPARATION CURVES OF A FLOW CLASSIFICATION APPARATUS OF FINITE LENGTH

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A simplified model of operation of a flow classification apparatus of finite length for the case of a low-concentrated suspension has been analyzed. An analytical and numerical analysis of the behavior of the separation function has been made. Particular emphasis has been placed on such characteristics of classification as the size and sharpness of separation. A self-similar solution for the concentration of particles in each phase has been obtained and compared to the numerical solution.

Introduction. Classification processes in technology are often based on the action of a force directed transversely to the flow of a suspension which is allowed to pass through the apparatus [1, 2]. The drift of particles which is caused by this force leads to their deposition on the wall. The drift velocity grows with particle size and, consequently, large particles are deposited faster than small ones. Therein, in essence, lies the classification effect. In the absence of accompanying phenomena (unequal inlet conditions for different particles and the complexity of flow including turbulence), the sharpness of separation would be absolute; all particles smaller than a certain critical d_j^* in size would escape from the apparatus, whereas large ones with $d_j > d_j^*$ would be trapped.

Below, we consider a schematic classifier of the type of hydrocyclones [1–4] for which one has currently developed quite reliable calculation methods and for which there are numerous computational formulas of empirical character [2, 4].

The approximate-analytical methods have been summarized in [2, 3, 5] and a numerical analysis of the processes in the apparatus has been performed in [6]. At the same time, a number of important aspects of the classification process have yet to be included into consideration. One can establish their significance by modeling the process in a formulation not involving numerous accompanying details which make the analysis more difficult. One such problem is the significance of the finiteness of the time of residence of particles in the apparatus.

In the present work, we analyze the influence of the nonequilibrium of the transport of particles on the characteristics of classification based on the ideas of a diffusion-turbulent model of the process in the apparatus [2, 7, 8]; according to this model, the transfer of particles to the exterior wall due to the centrifugal force is opposed by the diffusion flow caused by the high level of turbulence. It takes a certain time or, in terms of the stationary regime, a certain apparatus length to attain such an equilibrium of forces; clearly, this equilibrium does not have to exist for fairly short apparatuses. A similar formulation of the problem was proposed in [8] and was considered further in [9], but it was not adequately analyzed. A substantially close model was developed in [10]. In this work, the stochastic Kolmogorov–Fokker–Planck equation for the probability density of stay of a particle at a certain point of flow at a certain instant of time was used instead of the introduction of a forward diffusion particle flux. Clearly, it is possible and expedient to pass to the terms of diffusion theory, since this circumvents the difficulties of formulating boundary conditions. Unlike [10], we have abandoned the special properties of a centrifugal classifier, setting (following [8, 9, 11]) all the hydrodynamic velocities inside the apparatus to be coordinate-independent, which gives us an approach to obtaining analytical solutions.

Formulation of the Problem. *Classification-Apparatus Model.* We consider a classification apparatus (Fig. 1) into which a two-phase mixture flows with a constant velocity U on the left (liquid with solid-particle fractions) and

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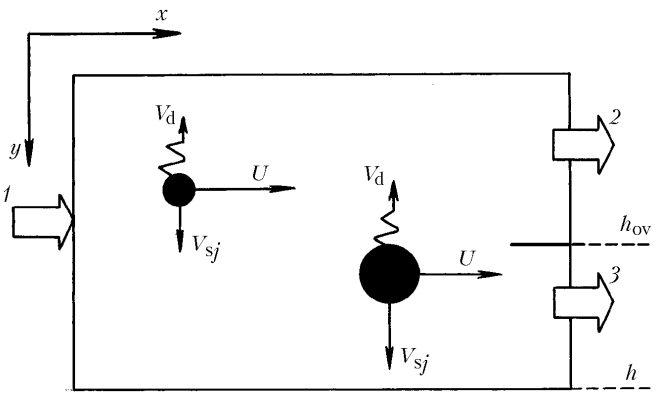


Fig. 1. Diagram of a classification apparatus: 1) inlet portion; 2) overflow launder; 3) underflow launder.

flows out on the right through upper and lower outlets. Furthermore, each particle of the *j*th fraction shifts downward (sediments) with a certain velocity V_{sj} which is the higher, the larger the particle of the *j*th fraction.

Let us assume that the sedimentation velocities of particles of different fractions are dependent just on the size d_j of this fraction's particles. Then the equation of variation in the particle concentration with space for each fraction together with the boundary conditions will be written in dimensionless form as follows:

$$\frac{\partial \theta_j}{\partial \xi} + \frac{\partial}{\partial \eta}\left(\mathrm{Pe}_j \theta_j - \frac{\partial \theta_j}{\partial \eta}\right) = 0 , \tag{1}$$

at the classifier inlet, we prescribe the initial concentrations of particles

$$\theta_j\Big|_{\xi=0} = 1 , \tag{2}$$

on the classifier wall, we set the condition of equality of particle fluxes to zero

$$\left(\mathrm{Pe}_j \theta_j - \frac{\partial \theta_j}{\partial \eta}\right)\Bigg|_{\substack{\eta=0 \\ \eta=1}} = 0 , \tag{3}$$

where

$$\theta_j = \frac{c_j}{c_{j0}} ; \quad \xi = \frac{xD}{h^2 U} ; \quad \eta = \frac{y}{h} .$$

Here the dimensionless parameter $\mathrm{Pe}_j = hV_{sj}/D$ characterizes the size of the *j*th fraction's particles. The coefficient of turbulent diffusion of particles *D* is considered to be a constant.

Integrating (1) from 0 to 1 with respect to η, we obtain the relation

$$\int_0^1 \theta_j (\xi, \eta)\, d\eta = 1 . \tag{4}$$

Determination of the Separation Curve. Let the width of the upper outlet be h_{ov}; then $h - h_{ov}$ is the width of the lower outlet. The ratio $S = h_{ov}/(h - h_{ov}) = \eta_{ov}/(1 - \eta_{ov})$ is called a split parameter whose value is usually about 10. Here we have $\eta_{ov} = h_{ov}/h$.

The rate of flow of the particles of the jth fraction through the lower and upper outlets respectively is usually determined by the following functions:

$$R_{\mathrm{un}j}(\xi) = \int_{\eta_0}^{1} \theta_j(\xi, \eta)\, d\eta\,, \tag{5}$$

$$R_{\mathrm{ov}j}(\xi) = \int_{0}^{\eta_0} \theta_j(\xi, \eta)\, d\eta\,. \tag{6}$$

They characterize the degree of separation of particles in each fraction. Large particles mostly pass through the overflow launder, whereas small ones pass through the underflow launder. The separation function determining the fraction of particles of a given fraction which is removed through the lower outlet is described, according to the separation model [2], by the following dependence:

$$T(\mathrm{Pe}_j, \xi) = \frac{R_{\mathrm{un}j}(\xi)}{R_{\mathrm{un}j}(\xi) + R_{\mathrm{ov}j}(\xi)}\,. \tag{7}$$

Using the fact that the longitudinal rate of feed of a suspension is presumably constant, we rewrite (7) with account for (4)–(6) in the form

$$T(\mathrm{Pe}_j, \xi) = \frac{\int_{\eta_0}^{1} \theta_j(\xi, \eta)\, d\eta}{\int_{0}^{\eta_0} \theta_j(\xi, \eta)\, d\eta + \int_{\eta_0}^{1} \theta_j(\xi, \eta)\, d\eta} = \int_{\eta_0}^{1} \theta_j(\xi, \eta)\, d\eta = \int_{\frac{S}{1+S}}^{1} \theta_j(\xi, \eta)\, d\eta\,. \tag{8}$$

A point separation function [2] can be another possible characteristic of separation. In this case, one uses certain average values of concentrations in the cross section ξ (for example, in outlets) in (8). Accordingly the separation function has the form

$$T(\mathrm{Pe}_j, \xi) = \left(1 + S\,\frac{\theta_{\mathrm{ov}j}(\xi)}{\theta_{\mathrm{un}j}(\xi)}\right)^{-1} \tag{9}$$

with certain average values of concentrations in these outlets. Thus, one frequently uses the computational formula

$$T(\mathrm{Pe}_j, \xi) = \left(1 + S\,\frac{\theta_j(\xi, 0)}{\theta_j(\xi, 1)}\right)^{-1}\,, \tag{10}$$

which differs from (9) in that it contains the values of concentrations on the apparatus walls.

Naturally, the expression with the values of concentrations taken at the center of the outlets

$$T(\mathrm{Pe}_j, \xi) = \left(1 + S\,\frac{\theta_j(\xi, \eta_{\mathrm{ov}}/2)}{\theta_j(\xi, (1+\eta_{\mathrm{ov}})/2)}\right)^{-1} \tag{11}$$

would be closer from the viewpoint of determination (7). The most important parameters characterizing the classification effect [2, 3] are

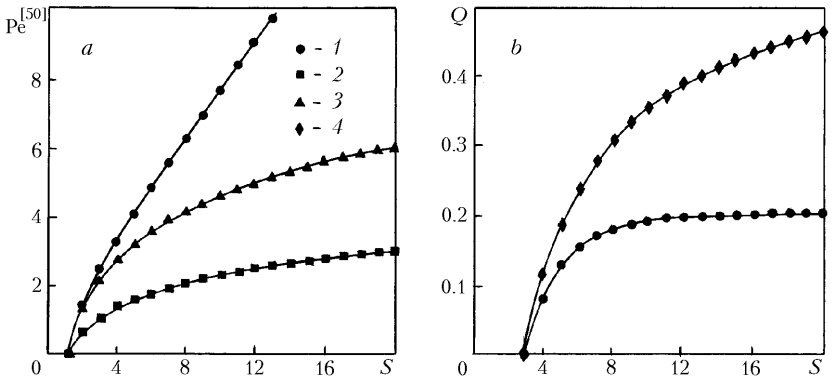


Fig. 2. $Pe^{[50]}$ (a) and sharpness of separation (b) vs. split parameter S in the methods of determination of the separation function according to: 1) (14); 2) (15); 3) (16); 4) (17).

(a) the parameter $Pe^{[50]}$ yielding the diameter of separation of particles 50% of which arrive at the lower outlet;

(b) the sharpness of separation Q, which is determined by the ratio of the particle diameters corresponding to values of the separation function of 0.25 and 0.75:

$$Q = \frac{Pe^{[25]}}{Pe^{[75]}} . \tag{12}$$

Analytical Solution of the Problem. *Total Equilibrium.* A concentration profile of the particles of each phase is established as the suspension moves along the channel. The final concentration profile is determined by the equilibrium of the sedimentation and diffusion rates.

The analytical solution of problem (1)–(4) for $\xi \to \infty$ is as follows:

$$\theta_j(\infty, \eta) = \frac{Pe_j}{\exp(Pe_j) - 1} \exp(Pe_j \eta) . \tag{13}$$

Accordingly, the separation function (8) computed based on (13) will have the form

$$T(Pe_j) = \frac{1 - \exp\left(-\dfrac{Pe_j}{1+S}\right)}{1 - \exp(Pe_j)}, \tag{14}$$

according to determination (10), it will be

$$T(Pe_j) = (1 + S\exp(-Pe_j))^{-1} \tag{15}$$

or, computed based on (11), it appears as

$$T(Pe_j) = (1 + S\exp(-Pe_j/2))^{-1} . \tag{16}$$

These three formulas are qualitatively the same, but their quantitative characteristics can strongly differ. If we use determination (15), then $Pe^{[50]} = \ln S$ and (16) yields a value twice as high. The use of (14) necessitates solution of a transcendental equation.

Figure 2a shows $Pe^{[50]}$ as a function of the parameter S in the case where different forms of determination of the separation function are used. Qualitatively the curves behave in the same manner but they strongly differ quantitatively. It is significant that all three models yield a uniquely growing dependence of $Pe^{[50]}$ on the split parameter S.

The sharpness of separation, calculated using (15) and (16), is a slowly growing function of S:

$$Q = \frac{\ln (S/3)}{\ln (3S)}. \tag{17}$$

The resulting curves are shown in Fig. 2b. According to the model (8), the sharpness of separation grows with S even more slowly than that following the logarithmic law (17).

Establishment of Equilibrium. For apparatuses of finite length, analytical solution of problem (1)–(4) by the Fourier method [12] yields

$$\theta_j (\xi, \eta) = \frac{\mathrm{Pe}_j}{\exp (\mathrm{Pe}_j) - 1} \exp (\mathrm{Pe}_j \eta) + 2\mathrm{Pe}_j \exp (\mathrm{Pe}_j \eta/2) \sum_{n=1}^{\infty} \left[1 + (-1)^{n+1} \exp (\mathrm{Pe}_j/2)\right] \times$$

$$\times \frac{\cos (\pi n \eta) + (\mathrm{Pe}_j/2\pi n) \sin (2\pi n)}{(\pi n + \mathrm{Pe}_j/4\pi n)^2} \exp\left[-(\pi^2 n^2 + \mathrm{Pe}_j^2/4)\, \xi\right]. \tag{18}$$

For fairly large ξ, confining ourselves to the first term of the sum in (18), we obtain the self-similar solution

$$\theta_j (\xi, \eta) = \theta_j (\infty, \eta) + F_j (\eta) \exp\left[-(\pi^2 + \mathrm{Pe}_j^2/4)\, \xi\right], \tag{19}$$

where

$$F_j (\eta) = 2\mathrm{Pe}_j \exp (\mathrm{Pe}_j \eta/2) \left[1 + \exp (-\mathrm{Pe}_j/2)\right] \frac{\cos (\pi\eta) + (\mathrm{Pe}_j/2\pi) \sin (2\pi)}{(\pi + \mathrm{Pe}_j^2/4\pi)^2}.$$

Based on (19), we take the following expression for concentration:

$$\theta_j (\xi, \eta) = \theta_j (\infty, \eta) + (1 - \theta_j (\infty, \eta)) \exp\left[-(\pi^2 + \mathrm{Pe}_j^2/4)\, \xi\right], \tag{20}$$

which corresponds to the relaxation-type equation

$$\frac{\partial \theta_j}{\partial \xi} = -\left(\pi^2 + \mathrm{Pe}_j^2/4\right)\left(\theta_j (\xi, \eta) - \theta_j (\infty, \eta)\right)$$

with condition (2) at the apparatus inlet. The self-similar solution (20) is quite close to the exact solution for $\xi > 0.1$, as a comparison to the numerical calculation shows.

We find an expression for the characteristic relaxation length, setting $(\pi^2 + \mathrm{Pe}_j^2/4)\xi = 1$. Here we have

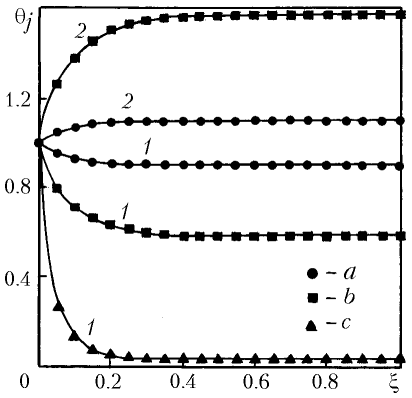
$$\xi_{\mathrm{r}j} = \left(\pi^2 + \mathrm{Pe}_j^2/4\right)^{-1}. \tag{21}$$

In particular, from (21) for $\mathrm{Pe}_j = 1$ we obtain that $\xi_{\mathrm{r}j} \approx 0.1$.

Formula (21) in dimensional variables has the following form:

$$\frac{U}{x_{\mathrm{r}j}} = \pi^2 \frac{D}{h^2} + \frac{V_{\mathrm{s}j}^2}{4D}. \tag{22}$$

In (22), we can single out the characteristic times of the process of classification, namely, the diffusion time ($t_{\mathrm{d}} = h^2/D$), the sedimentation time ($t_{\mathrm{s}j} = h/V_{\mathrm{s}j}$), and the time of residence of particles in the zone where their concentration is established ($t_{\mathrm{r}j} = x_{\mathrm{r}j}/U$); this time is determined by the relation



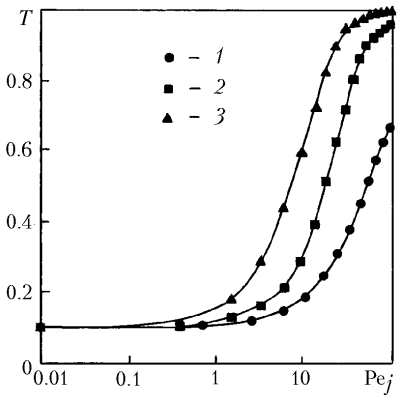


Fig. 3. Concentration of particles on the walls of the apparatus vs. its length ξ for different values of Pe_j: a) Pe_j = 0.2; b) 1.0; c) 5.0; 1) η = 0; 2) 1.

Fig. 4. Separation function vs. Pe_j (S = 9): 1) ξ = 0.01; 2) 0.03; 3) 5.0.

$$\frac{1}{t_{rj}} = \frac{\pi^2}{t_d}\left(1 + \frac{t_d^2}{4t_{sj}^2}\right).$$

It is clear from (21) that the relaxation length decreases with increase in the parameter Pe_j. Since higher Pe_j values correspond to larger particles (with a higher settling rate), the concentration of such particles is established at a shorter distance from the apparatus inlet than that in the case of small particles.

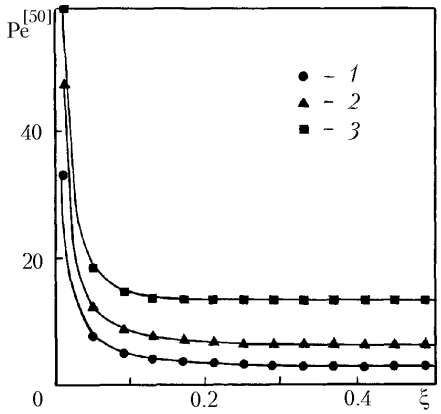
Numerical Analysis. The main conclusions drawn analytically can be substantiated and supplemented with the data based on numerical calculations in which Pe_j values varied between 0.01 and 100.

Figure 3 shows the pattern of establishment of the concentration of particles of different size (characterized by the value of Pe_j) on the opposite walls of the apparatus. The particle concentration decreases at the upper boundary of the classification apparatus (η = 0) and increases at the lower boundary (η = 1). The larger the size of a particle, i.e., Pe_j, the higher its settling rate; consequently, the influence of diffusion effects is small and particles sediment downward more quickly. As the particle size decreases, the settling rate becomes lower and the influence of the diffusion effect becomes stronger, because of which the concentration gradients of small particles across the apparatus disappear. As the analytical solution of (21) shows, the length on which the value becomes steady-state is dependent on the value of the Péclet number. The larger the Pe_j, the shorter the distance to the apparatus inlet at which the steady-state value of the particle concentration is attained, which is confirmed by Fig. 3.

In numerical solution of the problem, we have obtained the separation function from formula (8) as a function of Pe_j (Fig. 4) for different ξ values. As ξ increases, an increasing portion of the particles of each fraction is removed through the underflow launder of the separation apparatus. Only the largest particles (with $Pe_j >> 100$) can pass through this launder when the apparatus is very short (ξ = 0.01).

Here and in what follows, we use the determination of the separation function according to formula (8) to plot $Pe^{[50]}$ as a function of ξ (Fig. 5). It is clear from the figure that the separation diameter decreases with increase in ξ on the initial portion of the $Pe^{[50]}(\xi)$ curve, i.e., the value of the parameter Pe_j (and of the particle size), which corresponds to the fraction of particles half-settled below the line of the outlet, decreases with increase in the apparatus length. With further increase in ξ, the value of $Pe^{[50]}$ becomes constant, since the settling of smaller particles is counterbalanced by diffusion. The value of $Pe^{[50]}$ increases with split parameter S when the dimension of the underflow launder decreases relative to the dimension of the overflow launder.

The form of the dependence of the sharpness of separation on the apparatus length ξ for different values of the parameter S, which has been determined from formula (12) using the concentration fields obtained numerically and based on the self-similar solution (20), is shown in Fig. 6. The variations for low values of ξ are natural, since the self-similar solution (20) is correct only for fairly high ξ values. The value of the sharpness of separation Q drops with increase in ξ, except for the case of short apparatuses. The form of the function $Q(\xi)$ is determined by



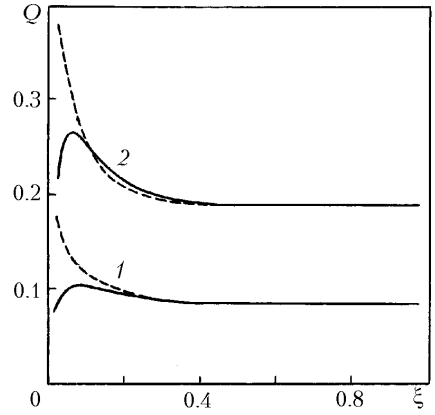


Fig. 5. Influence of the value of the split parameter S on $Pe^{[50]}$ as a function of the apparatus length ξ: 1) $S = 4$; 2) 9; 3) 19.

Fig. 6. Sharpness of separation, calculated numerically (solid curves) and based on the self-similar solution for concentration (20) (dashed curves) vs. apparatus length for different values of S: 1) $S = 4$; 2) 9.

the character of establishment of $Pe_j^{[25]}$ and $Pe_j^{[75]}$. The concentration distribution of large particles is established much faster than that of small particles (see (22)). Therefore, on the initial portion of the apparatus, $Pe_j^{[75]}$ drops to its steady-state value much faster than $Pe_j^{[25]}$ with increase in ξ. Accordingly, $Q \propto 1/ Pe_j^{[75]}$ grows. Once the concentration of relatively large particles has been established, $Pe_j^{[75]}$ begins to slowly drop with growth in ξ because of the counteraction of diffusion. In so doing, $Q \propto 1/Pe_j^{[75]}$ also drops down to its limiting value. It follows from Fig. 6 that the quality of separation of particles in the apparatus becomes worse with decrease in the parameter S, since the lower outlet of the apparatus increases and, consequently, the portion of particles of each fraction which is removed through this outlet grows.

From the results (presented in Figs. 5 and 6) of the numerical calculation, we can draw the following conclusion: on the one hand, to attain a lower value of $Pe^{[50]}$ (Fig. 5), as is often important in practice, one should use a fairly long classification apparatus; on the other, one must use a shorter apparatus to obtain the maximum sharpness of separation (Fig. 6).

Conclusions. Sedimentation and diffusion fluxes in a flow apparatus counterbalance each other at a distance from the outlet, shorter for large particles than that for small ones. A self-similar particle-concentration profile passing to a limiting one (dependent just on the distance to the walls) is established at a certain distance from the apparatus inlet. The difference in the quantitative characteristics of the separation function is substantially dependent on the method of its computation.

The separation size and the sharpness of separation grow with split parameter. The separation curve shifts to the region of smaller particles with elongation of the apparatus. The quality of separation drops, except for very short apparatuses.

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NOTATION

c_j, concentration of particles of the jth fraction, kg·m^{-3}; d_j, size of a particle of the jth fraction, m; D, coefficient of diffusion of particles, m·sec^{-2}; h, height of the classification apparatus, m; h_{ov}, height of the overflow launder, m; n, ordinal number of the term in the Fourier sum; Pe_j, Péclet number for particles of the jth fraction; $Pe^{[25]}$, $Pe^{[50]}$, and $Pe^{[75]}$, Péclet numbers obtained from values of the separation function of 0.25, 0.50, and 0.75 respectively; Q, sharpness of separation; R_{unj} and R_{ovj}, portion of the jth fraction's particles escaping through the lower and upper outlets of the apparatus; S, split parameters; t, time, sec; T, separation function; U, rate of feed of a two-phase mixture

to the apparatus, m·sec^{-1}; V_{sj}, sedimentation rate of a particle of the jth fraction, m·sec^{-1}; x and y, coordinates along the abscissa and ordinate axes, m; η, dimensionless coordinate along the ordinate axis; θ_j. dimensionless concentration of the jth fraction; θ_{unj} and θ_{ovj}, average value of the dimensionless concentration of the particles of the jth fraction respectively in the lower and upper outlets; ξ, dimensionless coordinate along the abscissa axis (along the apparatus length); ξ_r, dimensionless characteristic relaxation length. Subscripts: un and ov, lower (underflow) and upper (overflow) outlets; 0, at the apparatus inlet; j, fraction No.; d, diffusion; s, sedimentation; r, relaxation (time of residence of particles in the zone where the concentration is established); *, critical.

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